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ABSTRACT

This document is one of six which set forth the mathematics components of the Project SEARCH Articulated Curriculum developed by the Utica (New York) City School District. Each volume deals with a broad area of mathematics and lists objectives related to that area for all grades from K through 12. Each objective listed is described first in general terms and then in terms of specific skills which students should exhibit. This volume addresses techniques of solving problems throughout the curriculum. (SD)

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Utica City School District

Articulated Curriculum

Project Search

1975

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FORWARD

This Articulated Curriculum is being printed and bound in this manner to provide for on-going revision. This also serves as evidence of work completed during Phase III of Project SEARCH.

MATHEMATICS

K - 12

Problem Solving

ACKNOWLEDGEMENTS

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PROJECT SEARCH

ARTICULATED CURRICULUM

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August 1975

MRS. ROSE DANELLA and NORMAN I. SIEGEL, both former Board Members deserve special mention for all their efforts on behalf of Project SEARCH.

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MATHEMATICS	PROBLEM SOLVING
Equation Solving	**Grade K**
The student will know:	The student will:
- the relationship of numerals to the number sentence.	- substitute numerals and pictorial representations found in problems.
that a problem has a solution.	- orally list solutions.
- the concept of larger and smaller.	- compare and contrast concept of larger and smaller.
- the meaning of problem solving vocabulary.	- arrange materials and orally identify "on-under," "long, longer, longest," more than, less than, etc.
- that a graph represents comparisons.	- construct a bar graph.
	Grade 1
- that many Math problems will arise from classroom activities.	- explore problems based on classroom activities.
- that different problems require different operations.	- combine separate objects as they relate to operations.
- the meaning of problem solving vocabulary.	- orally identify "on-under" "long, longer, longest" "more-than" "less-than" using various materials.
- that number patterns are used in problem solving equations.	- identify number patterns in problem solving equations.
Graphing	
- that graphs represent comparisons.	- construct simple graphs.
Equation Solving	**Grade 2**
- that word problems arise from classroom activities.	- orally solve problems based on classroom activities.
- that numbers make patterns.	- demonstrate understanding of number patterns.

Equation solving	**Grade 2**
- that word problems can be translated into number sentences.	- translate word problems into number sentences.
- that number lines can be used to translate word problems into number sentences.	- use number lines to solve problems
- that problems can be solved using mental computation.	- solve word problems orally.
- the meaning of properties of order and grouping.	- be given a problem regarding number order and will demonstrate understanding by finding the correct solution.
Graphing	
- that graphs are used for comparison.	- construct and interpret simple graphs.
Equation Solving	**Grade 3**
- that open sentences can be used for problem solving.	- convert word problems into number sentences.
- that problems can be solved through a series of steps.	- solve problems using a series of steps using basic operations.
- that a graph is used for comparison.	- solve word problems by using a number line, line graph, picture graph, and bar graph.
Equation Solving	**Grade 4**
- that estimation before computation and determination of enough information is necessary before solving problems of comparative relationships and averages.	- estimate before and after computation. solve problems of comparative relationships. solve problems of finding averages. solve verbal problems involving reading--no numbers, unneeded numbers, and not enough information.

Equation Solving	**Grade 4**
- that word problems can be translated into number patterns.	- identify correct number patterns from various problems.
Graphing	
- that word problems can be translated into graphs.	-interrupt, determine, construct a given problem into a line graph or bar graph.
Equation Solving	**Grade 5**
- the proper steps in writing an equation of a given problem.	- write an equation in two steps or more to solve the problem.
- the meaning of the Associative Property and Commutative Property.	- write more than one equation to solve one problem using the associative and commutative property.
- the meaning of an indirect number sentence.	- determine the necessary parts of an indirect number sentence.
Graphing	
- that graphs show visual representatives of facts and figures.	- construct a bar graph and line graph using vertical and horizontal scales and label the vertical and horizontal scales correctly.
- that there are various types of graphs.	- select the correct graph to solve the problem.
Equation Solving	**Grade 6**
- that a direct sentence is one where the answer is missing while an indirect sentence has other parts missing. A closed sentence is either true or false while an open sentence is neither true nor false and contains a place holder. (14-5=3x6)	- solve direct, indirect, closed and open sentences from word problems.
- that a word problem can be translated into a number sentence using a parentheses to indicate the order of operation.	- solve 3-step word problems using parentheses to write number sentences.

Equation Solving

- that a ratio shows the correspondence between the numbers of two sets (3 to 5 or 3:5 or $\frac{3}{5}$)
- that operations can be performed on integers to solve problems. (e.g. temperature)

Graphing

- that graphs can be used to visually represent data in a convenient and organized manner.
- the need to find only the essential information in a problem.
- the means of arriving at a suitable number sentence (equation) for the solution of a verbal problem.
- the algorithm necessary to the solution of various catagories of verbal problems.
- that the solution to some problems requires but a single operation, while other problems require more than one operation to attain the solution.
- that some problems by their nature lend themselves to the use of a proportion to attain a solution.
- how to translate various kinds of problems such as coin problems, age problems, and mixture problems into number sentences in order to attain a solution.

Grade 6

- solve word problems using ratios.
- solve word problems involving negative integers.
- plot information on line and bar graphs.

Grade 7 - 8

- discriminate between the necessary and extraneous information in a problem situation and list only the information pertinent to the problems solution.
- translate the essential information of a problem into an equation using word clues such as:
  "and" + "is" =
  "difference" - "is not" ≠
  "product" x
- solve a series of verbal problems given the equation.
- identify from a list of problems those problems whose solution require but one operation and those problems which require more than one operation to attain the solution.
- translate a series of problems in which proportions may be utilized into number sentences (equations).
- solve a series of problems such as coin problems, age problems, and mixture problems.

	Grade 7 - 8
- certain operations take precedence over others (order of operations).	- solve a series of number sentences (equations) following correct operational order.
U.E. Foundations	Grade 9
- the method of solving an arithmetic fractional equation.	- solve an arithmetic fractinnal equation.
- the method of solving an arithmetic equation using whole numbers.	- solve an arithmetic equation involving whole numbers.
- the method of solving the three basic percentage problems.	- solve the three basic percentage problems
- how to solve equations involving: one operation; two operations; combinations of like terms; variables on opposite sides of equal sign; fractional equations; quadratic equations; systems of equations.	- solve equations involving: one operations; two operations; combination of like terms; variables on opposite sides of equal sign; fractional equations; quadratic equations; systems of equations.
- how to solve word problems involving: number; investment; digit; mixture; motion; age; work; consecutive integers.	solve word problems of this type: number; investment; digit; mixture; motion; age; work; consecutive intergers.
- how to graph a point on a coordinate system given the coordinates of the point.	- graph the point given the coordinates of the point.
- how to find the coordinates given a point on the coordinate system.	- give the coordinates of a point on a coordinate system.
- how to graph the solution set of linear equa- and inequalities.	- find the solution set oflinear equations and inequalities by graphing.
- how to find the solution set of a system of equation and inequalities by graphing.	- find the solution set of a system equations and inequalities by graphing.

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Computer Math

Equation Solving

	Grade 9 - 12
- that algorithms are needed to solve problems.	- solve a problem using an algorithm.
- that various programming techniques (skills) are used by the programmer in order to solve a problem.	- develop one program which utilizes a programming technique.
- that computer solutions should always be checked in order to insure that the solution is correct and reasonable.	- analyze a computer solution in order to determine its "reasonableness."
- that there are a variety of techniques for debugging a program.	- debug a given program using a technique learned.
- that a graphic (visual) presentative of any program can be done trhough flowcharting.	- draw a flowchart for a given problem.
- that a computer can be programmed to make drawings or to draw curves on a coordinate axis system.	- develop a program which instructs the computer to make a drawing.

	Grade 10
- angle relationships involving supplementary, complementary, vertical, acute, right, obtuse, etc. angles.	- solve a first degree equation involving relationships between the measure of angles in the given problem.

Advanced Foundations

	Grade 10 - 12
- the different types of equations.	- a. solve a linear equation in one variable. b. solve a proportional equation.

- a. the quadratic equation $ax^2 + bx + c = 0$
  b. the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  c. the discriminant.
  d. the relationship between roots and coefficients.
  e. the quadratic inequalities.

-a. solve the quadratic equation by factoring.
b. solve a quadratic equation using the quadratic formula
c. use the discriminant in determing the nature of the roots of a quadratic equation.
d. derive the quadratic equation from the roots and coefficients.
e. solve quadratic inequalities.

- the following Laws of Logarithms:
  a. $\log ab = \log a + \log b$.
  b. $\log a/b = \log a - \log b$.
  c. $\log a^n = n \log a$.
  d. $\log \sqrt[n]{a} = 1/n (\log a)$

- evaluate the following expression by using the laws of logarithms:

$$x = \sqrt{\frac{(8.4)\quad(7.3)}{(\text{s/n}\ \ 57^\circ\ 10')\quad(3)}}$$

- the method of interpolation of numbers to 4-figure accuracy and values of functions of angles to the nearest minute.

- interpolate and compute the values of N to the nearest hundredth using logarithms.

$$N = \frac{(5.124)\quad\sqrt{\cos 50^\circ\ 13'}}{\sqrt[3]{7.298}}$$

- solve exponential equations of the type
$4^x = 8$, $3^{x+1} = 27^x$, $N\ 1/3 \cdot N^{2/3} = 4^2$

- the following laws of exponents:
  a) $x^a \cdot x^b = x^{a+b}$
  b) $x^a \cdot y^a = (xy)^a$
  c) $x^a \div x^b = x^{a-b}$, $(a, b, x \neq 0)$
  d) $(x^a)^b = x^{ab}$

- the law of sines and law of cosines

- a. apply the law of sines and law of cosines to verbal problems.
  b. determine the area of triangles using the law of sines.

- the basic trigonometric identities.

- a. derive these identities.
  b. use them in the proofs of other identities.

- the trigonometric identities of the single angle and the double angle.

- that inequalities may be true for all, some, or no elements of the replacement set.

- the definition of absolute value.

- the definition of inverse trigonometric form.

PROBABILITY & STATISTICS

Introduction

- the definition of probability.

- that a sample space may be made to show complementary events.

- that a sample space may be made to show mutually exclusive events.

- that a sample space may be made to show independent events.

- the formula for the probability for the union of two sets.

- the formula for complementary events of a sample space.

Grade 11

- solve trigonometric equations involving two or more functions.
- express any function of an angle in terms of any other function of that angle.

- solve three inequalities for a particular value.

- solve inequalities algebracially, using the definition of absolute value.

- express the roots of a trigonometric equation in inverse trigonometric form.

Grade 11 - 12

- apply the definition of probability to answer a question.
- make a sample space showing complementary events.

- make a sample space showing mutually exclusive events.

- make a sample space showing independent events.

- apply the formula for the union of two sets of a sample space to a specific problem.
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- apply the formula for complementary events to a specific problem.
  $P(A) = 1 - P(\overline{A})$

Introduction	Grade 11 - 12
- the formula for the intersection of two sets of a sample space.	- apply the formula for the intersection of two sets of a sample space. $P(A \cap B) = P(A) \cdot P(B / A)$
- the definition of independent events.	- write the formula for independent events. $P(A \cap B) = P(A) \cdot P(B)$

Graphing	Grade 11 - 12
- the technique for making random selections	- collect data using random selection techniques.
- the technique for presentation of data.	- organize the data collected.
- that data may be presented by use of a "Frequency Table."	- make frequency table, using the data collected.
- that data may be presented by means of a Dot Frequency Diagram.	- make a Dot Frequency Table using the data collected.
- that data may be presented by use of a cummulative graph.	- construct a cumulative graph using the data collected.
- the technique for finding the quartiles using the cumulative graph.	- find the quartiles using the cumulative graph.
- the technique finding the percentiles using the cumulative graph.	- find certain percentiles using the cumulative graph.
- the technique for making a cumulative polygon.	- make a frequency histogram using graphed data.
- the techniques for misrepresentation of data.	- make a cumulative polygon using the data collected.
- that one may use the Monroe X,Y Plotter to prepare graphs.	- use given graphs to detect deceptive practices.
- that data may be presented by means of a Dot Frequency Diagram.	- construct graphs using the X,Y. Plotter.

	Grade 11 - 12
Equations	
- that Chefyshev's Theorem may be used to analyze data.	- apply Chefyshev's Theoren using collected data.
- that a binomial may be expanded.	- a. expand a binomial b. find and simplify a specific term of a binomial expansion.
- that the binomial expansion may be used to solve a probability problem.	- use the binomial expansion to solve a probability that a baseball player hits fewer than 4 homeruns in his next 10 trips to bat.
- that one may use the binomial distribution tables.	- solve a probability problem using the binomial distribution tables.
- that one may make histograms using the binomial distribution tables.	- make a histogram using the binomial tables.
- the use of the standard normal curve tables.	- use the standard normal curve tables to find the area under the curve between two abscissas.
- the techniques of changing a normal distribution to a standard normal distribution.	- change a normal distribution to a standard normal distribution.
Computer Programming	**Grade 12**
- the methods and techniques relating to writing computer programs that may be used in problem solving.	- apply the computer to problem solving of conic sections related types.
- the methods how to properly prepare a computer program flow chart.	- prepare input data from a given equation (quadratic) that typifies a specific conic section.
- the applications of computers to solving problems relating to conic sections.	- present this raw data to the computer, either manually (keyboard) or via punched cards.
- the preparation of a computer program that can apply to conic sections.	- operate the computer properly and secure a printout.

Grade 12

- the use of such programs to be put to actual use in problem solving.
- the proper interpretation of output results of the computer output data.

- interpret output results into meaningful algebraic format.
- utilize output data to construct graphs representative of the specific related conic section.

Grade 12 X

- the method of proving a problem by direct, indirect, and inductive techniques.
- the method of sketching the graph of a given relation
- the definition of permutation and combination.
- the meaning of random variables and probability distribution.
- binomial distribution.
- the method to determine the relative maximum, relative minimum and inflection points of a graph.

- solve a given problem using direct, inductive reasoning.
- graph $F(x) = 2 = x + 1$ when $x = 3=x=3$
- use the formula $Pr = \frac{n}{(n-r)}$ to find the solution $_{n}$
  for the given problem with 8 flags of different color, how many possible arrangements of 3 flags can be made.
- use the formula $C = {}_{nr}P_{r} = \frac{n}{r-(n-r)}$ to find the probability of drawing 4 clubs from a well-shuffled deck.
- find the probability distribution of X given: 2 dice, letting X be the number shown when (X-2, 3, ...12)
- find the probability that double heads will appear 4 or more times when 2 coins are tossed simultaneously six times.
- find the maximum value of:
  $P = (x,y_ : y = x - \frac{3.125x^2}{10^6}$

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A.P. Math	Grade 12
- the pllication of derivative formulae to verbal problems: extreme value problems; velocity and acceleration problems; related rates of change problems.	- apply the derivative formulae in solving verbal problems: extreme value problems; velocity and acceleration rpoblems; related rate problems.
- the application of integration formulae to verbal problems; area and volume problems; growth and decay problems; harmonic motion problems.	- apply the appropriate integration formulae in solving verbal problems: area, volume, growth and decay, and harmonic motion problems.
- the derivation of the derivative formulae: polynomial, trigonometric, exponential, logarithmic function; sum, product and quotient of function; composite functions; inverse trigonometric functions.	- apply the specific derivative formulae: polynomial, trigonometric, exponential, logarithmic functions; sum and product and quotient of functions; composite functions; inverse trigonometric functions.
- the mean vlue theorems for derivatives and integrals.	- apply the mean value theorems for derivatives and integrals.
- the Rolle's Theorem.	- apply Rolle's Theorem.
- the solution of differentive equations of the first degree.	- solve differential equations of the first degree.
(b) Graphing	Grade 12
- the techniques of graphing polynomial functions through the fourth degree.	- graph polynomial functions through the fourth degree.
- the techniques of graphing circular trigonometric functions.	- graph circular trigonometric functions.
- the techniques of graphing inverse trigonometric functions.	- graph inverse trigonometric functions.
- the techniques of graphing ellipses, circles, hyperbolae, and parabolas.	- graph ellipses, circles,hyperbolae and parabolas.

(b) Graphing	Grade 12
- the techniques needed to graph an equation and indicate revolution about a line parallel or perpendicular to the x axis.	- graph an equation and illustrate revolution about a line, parallel or perpendicular to the x axis.
- the graph of a problem to illustrate Rolle's Theorem.	- graph a problem to illustrate Rolle's Theorem.
- the techniques of graphing exponential and logarithmic equations.	- graph exponential and logarithmic equations.
- the techniques of analyzing an equation for asymptotes; x and y intercepts; symmetry; and extreme values before graphing the equation.	- analyze an equation and state asymptotes, symmetry; x and y intercepts; and extreme values before graphing the equation.
- the techniques of graphing, using polar coordinates.	- graph polar equations.